PHYSICS 536

Experiment 2: Frequency Response of AC Circuits

A. Introduction

The concepts investigated in this experiment are reactance, impedance, and resonance circuits. Many features of the scope will be used: including dual traces; differential inputs; and external triggering. Since this is the first experiment in which you have used the oscilloscope so a little extra care is appropriate. Refer to the General Instructions for the Laboratory for additional information. A brief synopsis of the concepts and equations needed for this experiment are presented below.

AC (alternating current) can be represented as a complex quantity. In the complex plane the vector representing current is referred to as a phasor. The magnitude of this vector is proportional to the current amplitude and the angle from the real axis is the phase. Voltage can also be represented in similar fashion.

1. **Reactance** Capacitive and inductive reactance are analogous to resistance and have units of ohms. Ohms law, $V = IR$, is an essential element in DC circuit analysis. If the concept of impedance is introduced, an analogous relationship still holds, $V = IZ$, where *Z* is impedance. *Z* is defined as $Z = R + jX$, where *X* is the reactance. *Z* is a complex quantity. Likewise, AC voltages and currents can be represented as complex quantities. As such, they have two independent components: amplitude and phase. Accordingly, $V = \cos\theta + jv\sin\theta$. The magnitude of the voltage is given by the relationship $V = \sqrt{(V_{real})^2 + (V_{imag})^2}$. The reactance of a capacitor is given by the formula
$$X_c = \frac{1}{\omega C}. \tag{1.1}$$
Reactive inductance is
$$X_L = \omega L \tag{1.2}$$

   Since impedance, voltage, and current are represented as complex quantities rather than using the cumbersome standard notation we utilize a shorthand notation in which the first quantity is the magnitude of the phasor and the second is the angle relative to the real axis. This polar notation for phasors is commonplace in electronics.

   For example, for a capacitor, $x_C = X_C\angle -90^\circ = \dfrac{1}{\omega C}\angle -90^\circ$.

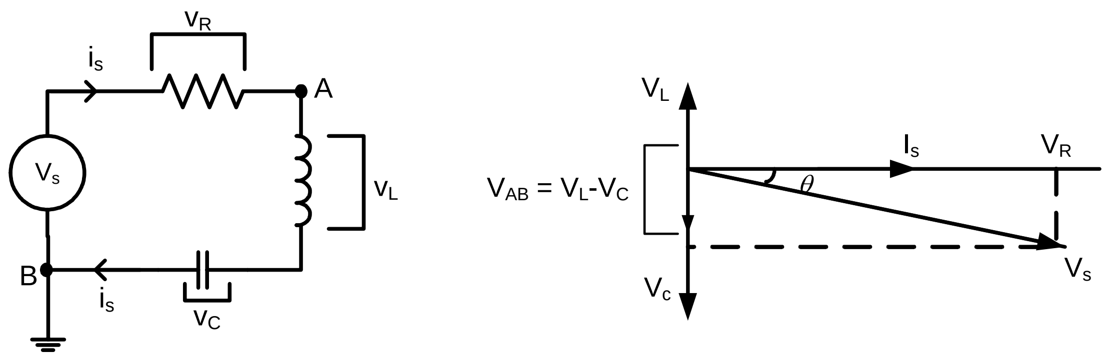
   Likewise, for an inductor, $x_L = X_L\angle +90^\circ = \omega L\angle +90^\circ$.

   Given an input voltage across a capacitor the current is given by
   $i = v / Z = v(\omega C)\angle +90^\circ$ so current leads voltage in capacitor.

Likewise, for an inductor $i = v/Z = \frac{v}{\omega L}\angle -90^{\circ}$ so current lags voltage in an inductor.

2. **RLC Circuit.** Phasors provide a simple way to visualize the relationship between current and voltage when R, C, and L are mixed in a circuit. The length of the vector represents the amplitude (i.e. peak-value) of the sine wave. Current and voltage, V_R, are in phase for a resistor, hence the voltage across a capacitor, V_C, and inductor, V_L, are both 90° out of phase with V_R in a series circuit. Kirchoff's voltage law requires that applied voltage, V_s is

Figure 1



equal to the sum of voltages across the three components. Although the three component voltages peak at different times the vector sum of the voltages across the three components sum to the input voltage

$$V = \sqrt{V_R^2 - (V_L - V_c)^2}\ . \tag{1.3}$$

The phase angle is given by

$$\phi = \arctan[(V_L - V_C)/V_R]\,. \tag{1.4}$$

At the resonance frequency f_o, X_C , and X_L are equal. $\Rightarrow 1/\omega C = \omega L$.

$$\omega_0 = \frac{1}{\sqrt{LC}}\,. \tag{1.5}$$

Since the reactances sum to zero both V_L and V_C must also sum to zero. At the resonance frequency V_R is maximum. The current in the circuit also is maximized since $I = V_R / R$. Since I is maximized, the magnitude of both V_C and V_L are individually at maximum, although 180^o out of phase with each other. When the frequency is not near the resonant frequency, the phasors representing V_C, and V_L do not cancel, so V_R is less for a given V_s.

It is convenient to do calculations in terms of impedance. For this RLC circuit the impedance is given $z = Z\angle\phi$ with

$$Z = \sqrt{R^2 + (X_L - X_C)^2} \tag{1.6}$$

and

$$\phi = \arctan(X_L - X_C)/R\,. \tag{1.7}$$

3. Parallel LC Circuit and practical inductors Long wires are used to wind real inductors, hence they have some resistance, r_s, that spoils the ideal character of series LC resonance; the impedance is equal to r_s, rather than zero, at resonance. The equivalent resistance in the parallel representation is,

$$r_p = \frac{(\omega L)^2}{r_s}\,.$$

An AC source driving a parallel inductor and capacitor as a practical matter resembles a parallel RLC circuit, where r_s and r_p are the resistances of the current source and inductor, respectively.

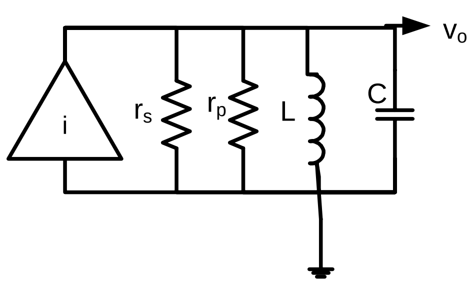


Figure 2.

The condition for resonant frequency is that the capacitive and reactive reactances sum to zero. This condition yields a resonant frequency of

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{1.8}$$

At this frequency the impedance is resistive and the voltage is maximum and given by

$$v_o = iZ = i(r_s \,||\, r_p)$$

The quality factor of any circuit can be defined as 2π times the energy stored per cycle divided by the energy lost or dissipated (usually heat) per cycle.

$$Q \equiv 2\pi \frac{W_S}{W_L} \tag{1.9}$$

For the parallel RLC circuit in Fig. 2 the Q is given by

$$Q = \frac{(r_s \parallel r_p)}{\omega L} \qquad (1.10)$$

B is the bandwidth of the circuit, *i.e.* the difference between the upper and lower frequencies at which the gain in the output voltage, v_o, has changed by 30%. In terms of the resonant frequency and the quality factory the bandwidth is given by

$$B = f_o / Q \qquad (1.11)$$

B. Inductors and Capacitors in Circuits

The reactance of an inductor, *L*, or capacitor, *C*, are investigated in this section.

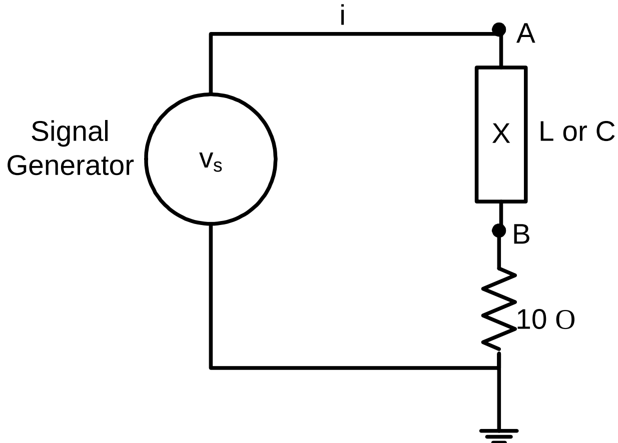


Figure 3.

The signal generator applies a sinusoidal voltage, v_s, across *L* or *C*. The 10 Ω resistor is included to measure the current in the circuit. The current across this resistor is given by $i = v_B / 10\Omega$. A small resistor is used so that the voltage, v_B, across it is negligible compared to the voltage across the capacitor or inductor. Accordingly, v_A is the voltage drop across the capacitor or inductor.

1 - Using f = 32 *kHz*, v_s = 10 *V* (p-p), C = 5 *nf*, measure v_B. Refer to GIL sections 5.4, 5.6, 5.7, and 5.8 for additional details. Use v_B to calculate the current, *i*, and use *i* and v_A to calculate X_C. Verify that the resistance is small compared the the reactance. Measure the relative phase between current and voltage (refer to GIL section 5.11 for further details). Internal triggering can be used. Recall that the scope probe attenuates the signal by a factor of 10.

2 - Repeat step 2 for an inductor with the following measurements f = 1 *MHz*, v_s = 10 V(p-p), *L* = 100 *μH*.

C. Practice Using Oscilloscope Triggering.

Connect the "output" of the signal generator to input 1 of the scope and the "1 volt" signal to input 2 of the scope. Set the generator for 10 *kHz*. Press the 1*V* amplitude button on the generator and turn the amplitude dial to 10. The output signal will be 3 *V*(p-p) sine wave because the buttons are labeled with the effective value. Press the channel 1 and 2 “menu” buttons to display both signals at the same time. The two traces should be separate on the face to face to avoid confusion. The traces are labeled with an arrow on the left side of the display. Set the trigger for rising slope on cannel 2. Press TRIGGER menu button. Select EDGE, RISING slope, CH2 source, AUTO mode, DC coupling. The trigger time is marked by an arrow at the top of the display. The trigger level is marked with an arrow on the right side of the display. Set the trigger level approximately half way between the peak values of the sine wave. Notice that the output and 1-volt sine waves are in phase.

a) In this step you observe the advantage of triggering on a constant signal when you want to observe changes in a signal. The output signal can be switched between sine wave and square wave. The 1-volt signal in channel 2 that is used to trigger the scope is not affected by this switch. The output square wave has the opposite polarity of the sine wave, *i.e.* the square wave is low when the sine wave is high. Switch between the sine and square wave because the trigger signal in channel 2 is not changing.

b) Next you will trigger from the signal that is changing to see that the phase relationship is not displayed correctly. Change the trigger source to channel 1. Then switch between the square and sine wave while observing channel 1. The sine and square wave appear to be in phase, *i.e*. both are high in the same part of the cycle. This is not the true relationship, as observed in part a). Since we are triggering on channel 1, both wave forms have a positive slope at the trigger point. Notice that the 1-volt signal in channel 2 is displayed in phase with the sine wave and out of phase with the square wave. Change the trigger between channels 1 and 2 until you understand the relationships. Ask the instructor for help if this illustration is not clear.

c) When you are observing changing signals on both input channels, you can not get a constant trigger from either of them. The scope provides a third “external trigger”. Move the 1-volt signal from channel 2 to external-trigger. Press TRIGGER menu and select EXT source. Switch between square and sine wave. The display in channel 1 will be the same observed in part a).

D. Representation of a Simple RC Circuit

In the following experiments use the vector representations of AC signals to help you visualize the effect of phase difference in the following circuit.

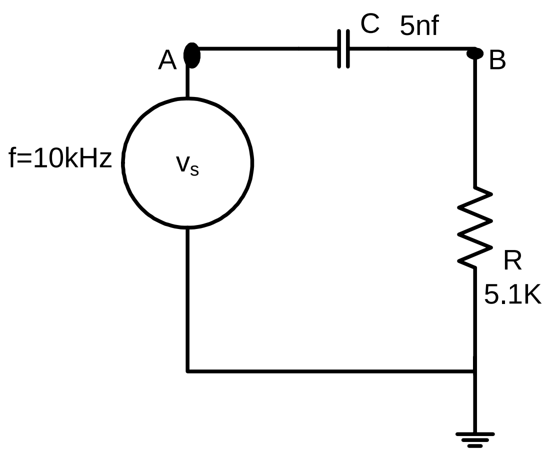


Figure 4

3 – As an exercise assume that the signal current is 1.67 mA (p-p). Calculate the voltage (p-p) across the capacitor, v_{AB}, and resistor, v_B. What is the phase between v_B and v_{AB}. Calculate the voltage, v_A (p-p), of the applied signal and the phase relative to v_B.

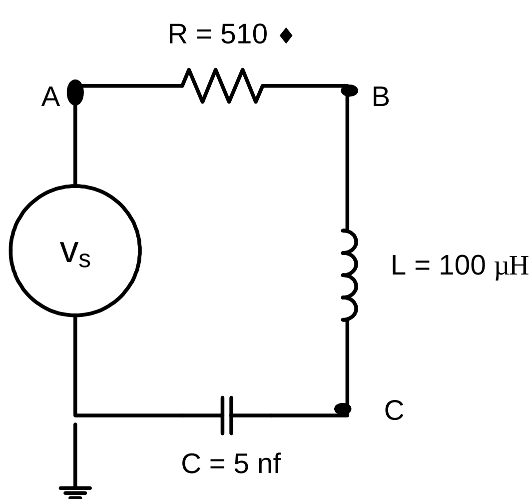
4 - Use a dual trace mode. Adjust the p-p value of the generator signal to be equal to the v_A calculated in exercise 2. Measure v_B and the phase between v_A and v_B. Remember the location of vertical grid mark where v_B reaches a peak because v_B will not be seen when you switch to differential mode. Switch the scope to differential horizontal sweep or triggering. The scope displays the difference between the A and B channels, *i.e.*, v_{AB}.

5 - Measure v_{AB} and the phase between v_{AB} and v_B.

E. Series RLC Circuit

In this section you will investigate the behavior of a series RLC circuit at the resonance frequency and away from resonance. Refer to Fig. 5.

Figure 5



6 - Measure the resonance frequency by observing when v_B in minimum. Measure the amplitude of v_C at this frequency and the phase between v_A and v_C. Explain how v_B can be minimum and how v_C can be very large at resonance. As part of the

laboratory report calculate the resonant frequency of the circuit using v_S = 5 *V* (p-p). Calculate v_B, v_{BC}, and v_C at resonance.

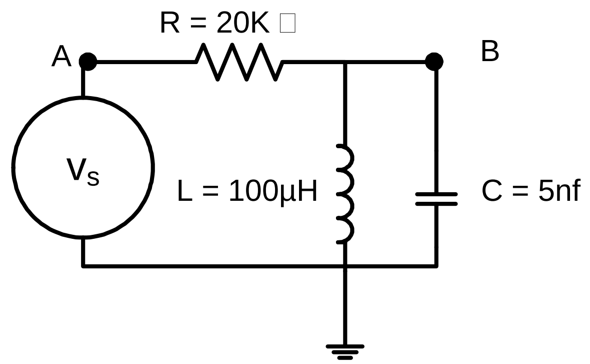
7 – Using v_S = 5 *V* (p-p), measure the peak-to-peak value of v_B and v_C, and the phase shifts between v_A and v_B, and between v_A and v_C.

8 – Set up a spreadsheet that calculates the reactance and impedance of this circuit, as a function of frequency, for the component values shown in the figure and v_S given in step 7. Graph the results, showing clearly the resonant frequency. Include this graph in the laboratory report. Which component dominates for $f = f_0$, $f < f_0$, and $f > f_0$? Explain your interpretation of the spreadsheet calculation.

F. Parallel RLC Circuit

In this section you will consider the impedance of a parallel RLC circuit as a function of frequency. The signal generator with a resistor in series is used as a current source.

Figure 6



9 - Apply Norton's theorem to the generator and resistor to obtain the equivalent current and resistance. Include these with your laboratory report.

10 - Observe the resonance frequency at which v_B is maximum. Use the observed v_B at resonance to calculate the resistance (r_s, r_p). Using this resistance calculate the effective quality factor, *Q*, of the circuit.

11 - Vary the frequency to measure the bandwidth, *i.e.*, the points at which v_B has dropped by 30% from the value at f_o. Use Q from step 10 to calculate the bandwidth and compare to that observed.

These calculations and measurements should demonstrate that the equivalent resistances (r_s and r_p) limit the maximum signal amplitude, v_B, that can be obtained from a parallel LC circuit for a given current. r_s and r_p also set a lower limit on the bandwidth that the circuit can provide.

Components

Resistors: 10Ω, 510Ω, 5.1Ω, 20K.
Capacitor: 5fn
Inductor: 100µH.

GROUND CONNECTIONS

uction

The issue of ground connections is one that will concern you again and again, in this and in other labs. Read this chapter carefully, and try to understand it as much as possible. Not everything in it may make perfect sense in the beginning; some of this material will become clearer as you gain laboratory experience. Nevertheless, it pays to have a preliminary understanding at this point. As you attempt to connect various instruments in future experiments, you may need to return to this chapter for advice.

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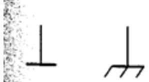
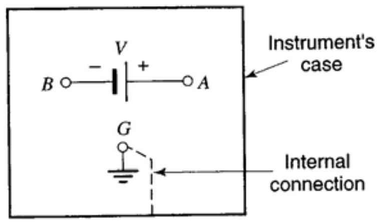
Lab instruments have terminals so that you can connect them to the circuits you are working with. For example, a "floating" power supply has plus (+) and minus (−) terminals. The voltage between them is a well-defined quantity, which you can set at will. However, the voltage between one of these terminals and a third point, such as the instrument's metal case (if it has one) or the case of another instrument, may not be well defined and may depend, in fact, on instrument construction and parasitic effects that are not under your control. Parasitic voltages can interfere with proper operation of the circuits you are working with, or can even damage them. Worse, in some cases they can cause an electric shock. To avoid such situations, the instruments have an additional terminal called the *ground,* often labeled GND or indicated by one of the symbols shown in Fig. 1; the use of this terminal will be explained shortly. The ground terminal may be connected to the internal chassis of the instrument (it is sometimes referred to as the chassis ground); to the instrument's metal case; and if the power cord of the instrument has three wires, to the ground lead of the cord's plug. When you plug in the instrument, this lead comes in contact with the ground terminal of the power outlet on your bench, which is connected to earth potential for safety and other reasons. In fact, other instruments on your bench, on other benches, or even elsewhere in the building may have their grounds connected to that same point, through the third wire of their power cords.



When you use a power supply with the output floating (i.e., with neither of the output terminals connected to ground), you get the situation shown in Fig. 2. The little circles indicate terminals for making connections to the instrument's output or ground. In the following discussion, V_{XY} will denote the voltage from a point X to a point Y. In Fig. 2, V_{AB} is the power supply's output voltage V, and it is well-defined. However, V_{AG} and V_{BG} are *not* well defined and can cause the problems already mentioned. To avoid this, you should strap one of the two output terminals to the ground terminal, as shown, for

Fig. 2



example, in Fig. 3(a). Now *all* voltages are well defined: $V_{AB} = V$, $V_{BG} = 0$, and $V_{AG} = V_{AB} + V_{BG} = V + 0 = V$. If we assume that V is a positive quantity, the connection in Fig. 3(a) develops a positive voltage at terminal A with respect to ground. If you happen to need a negative voltage with respect to ground instead, you would use the connection in Fig. 3(b). Here terminal B has a potential of $-V$ with respect to ground. In some power supplies, ground connections as shown in Fig. 3 are permanent, and you do not have access to them. In other power supplies, it is up to you to make such connections.

Connecting one grounded instrument to another

When more than one instrument or circuit with ground connections are used, one should think carefully. Consider the situation in Fig. 4, where it is attempted to connect the output of one instrument to the input of another. For example, instrument 1 can be a function generator, discussed in Experiment 3. Instrument 2 can represent an oscilloscope, or an oscilloscope probe, also discussed in Experiment 3. At first sight, the connections shown seem to be correct. However, there is a *big* problem. Although not apparent from Fig. 4, the ground terminals are connected not only to the instrument cases *but also to the common ground of the power outlet on the bench* (through the ground pin on the power plug, as explained earlier). Making these connections explicit, we have the situation shown in Fig. 5. It is now clear that the second instru-

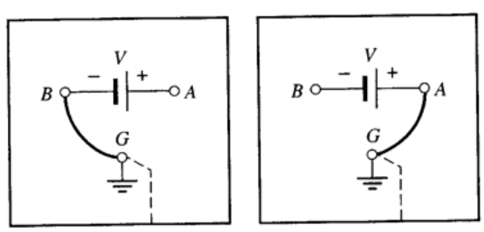


Fig. 3 (a) (b)

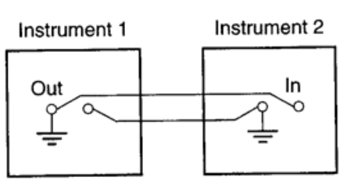


Fig. 4

ment's ground connections short the first instrument's output across *CD* (i.e., they place a short circuit across it; you can trace this short circuit along the path *CIHGED*). Not only will this prevent a voltage from being developed at that output, but also it can damage the instrument. The problem is solved if the connection between the two instruments is modified, so that *instrument ground is connected to instrument ground*, as shown in Fig. 6.

At this point, one may wonder what the connection marked x is needed for in Fig. 6, given that the two ground terminals are connected together anyway through the power cables, as shown by the heavy lines. The answer is that there may not always be a ground terminal on the power plug, and even if there is one, it may not be reliable; although ideally all ground terminals on power receptacles should be at the same potential, they sometimes are not. In addition, the long ground wires (*IH* and *GH* in Fig. 6) may act as antennas, picking up interference. To be safe, then, use a short connection such as x between the ground terminals of the two instruments.

A final word of caution: Since an instrument's case is in contact with ground connections, you need to be sure that cables and devices do not accidentally come into contact with it. If this happens, malfunction or damage can occur.

These guidelines will be sufficient for the purposes of this lab. Grounding is actually a complicated issue, and you should not expect the simple practice discussed above to be adequate in all situations. As you gain experience, you will obtain a better feel for grounding practices.

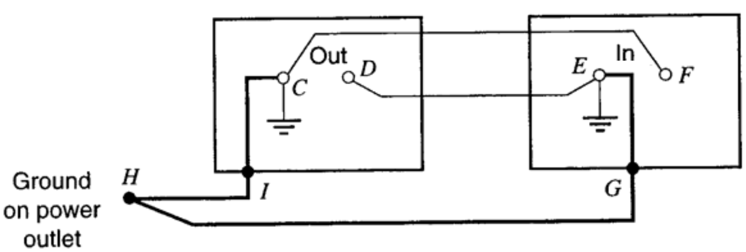


Fig. 5

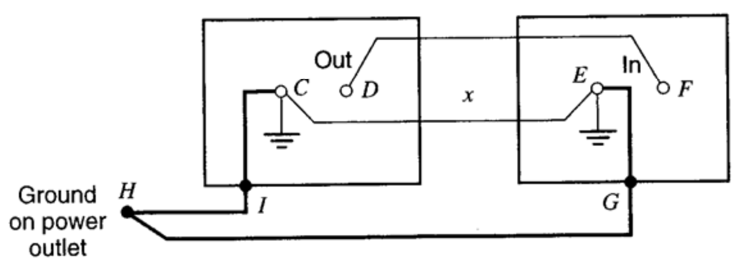


Fig. 6